

# Gravitational waves in a stringlike fluid cosmology

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## Abstract

The coupling of a stringlike fluid with ordinary matter and gravity may lead to a closed Universe with the dynamic of an open one. This can provide an alternative solution for the age and horizon problems. A study of density perturbations of the stringlike fluid indicates the existence of instabilities in the small wavelength limit when it is employed a hydrodynamic approach. Here, we extend this study to gravitational waves, where the hydrodynamical approach plays a less important role, and we argue that traces of the existence of this fluid must be present in the anisotropies of the cosmic background radiation.

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# 1 Introduction

Exotic kinds of matter has been considered in many different contexts in cosmology. When the exotic matter has an equation of state of the type  $p = -\frac{1}{3}\rho$  ( what is currently called  $K$ -fluid ), it is possible to obtain a closed Universe (  $k = 1$  ) having the dynamics of an open Universe[1, 2]. This fact leads to two interesting consequences: since the Universe is closed, there is a solution for the horizon problem without the employment of an inflationary phase; moreover, if we are now in a phase dominated by this exotic fluid, the scale factor behaves as  $a \propto t$ , and the Universe is older in comparison with the prediction of the Standard Cosmological Model, by a factor  $\frac{3}{2}$ . Since the evaluation of the age of the Universe coming from the observational measure of the Hubble parameter is dangerously near the evaluated age of globular clusters, such an older Universe can have attractive features. These models can be freely called a "coasting" Universe, with a stringlike fluid that dominates asymptotically the dynamic.

In preceding works, we have studied the behaviour of density perturbations in an Universe dominated by this exotic fluid[3, 4] using a hydrodynamic approach. We found that there exist strong instabilities in this fluid in the small wavelength limit that can be reflected in the anisotropy of cosmic background radiation deduced from density perturbations. This phenomenon occurs if we consider the matter content of the Universe being just this exotic fluid or if it is in presence of ordinary matter, but with no direct interaction between them. This contradicts some previous claims that these fluid would have no consequence at this level[5].

Here, we extend that study to gravitational waves since they can also impinge traces in the anisotropy of cosmic microwave background. The study of gravitational waves in this scenario has many motivations. The knowledge of the evolution of tensorial modes in an expanding Universe is an interesting subject by itself. But there exists some others physical motivations. Our previous study[3, 4] is in some sense limited, since it consider a hydrodynamical model only, and not fundamental fields, as it should be done if we are more rigorous[6]. On the other hand, in the equation of gravitational waves the matter content is not present directly, but only through the behaviour of the scale factor. This is

not case for density perturbations, where the type of matter we consider is determinant in the behaviour of the perturbed quantities. So, even if we employ here again a hydrodynamic approach, in order to have analytical expressions, our results are characteristics of models where the scale factor behaves asymptotically as  $a \propto t$ , that is, dominated by the  $K$ -fluid, and in this sense our final conclusions can be extended to more realistic models.

The relative contribution with respect to density perturbations is still controverse [7, 8]. However, the study of the polarization of the CMB photons may permit to separate the contributions of gravitational waves and density perturbations, even though this is yet out of observational limit[9]. In the present work we will indicate qualitatively how the presence of that exotic fluid must be reflected in the anisotropies of the CMB radiation, paying attention to the contribution of the gravitational waves.

## 2 Background solutions and perturbed equations

We consider here a model where ordinary matter is coupled to an exotic fluid, which we generally call *stringlike fluid* or  $K$ -fluid. They do not interact directly between themselves, but only through the geometry. The Einstein's equations can be written as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}^{(m)} + \kappa T_{\mu\nu}^{(s)} , \quad (1)$$

$$T^{(m)\mu\nu}_{;\mu} = 0 , \quad (2)$$

$$T^{(s)\mu\nu}_{;\mu} = 0 . \quad (3)$$

In the above expressions,  $(m)$  designates ordinary matter, while  $(s)$  designates the string-like fluid. Both fluid can be written in a perfect fluid form,  $T^{\mu\nu} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu}$ . Inserting the Robertson-Walker metric  $ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1+k^2r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$ , we obtain the following equations of motion,

$$3\left(\frac{\dot{a}}{a}\right)^2 + \frac{3k}{a^2} = 8\pi G(\rho_m + \rho_s) , \quad (4)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_s - 3\alpha\rho_m) , \quad (5)$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(1 + \alpha)\rho_m = 0 , \quad (6)$$

$$\dot{\rho}_s + 2\frac{\dot{a}}{a}\rho_s = 0 . \quad (7)$$

In these expression  $k$  is the curvature of the spacelike section,  $\rho_m$  is the energy density of the ordinary matter,  $\rho_s$  is the energy density of the stringlike fluid, and  $p_m = \alpha \rho_m$ . Since  $\rho_s \propto a^{-2}$ , we can define in equation (4) an effective curvature term that can be positive, negative or zero even if we fix  $k = 1$ . The resulting equation can be written as

$$\frac{\dot{a}^2}{a^2} - \frac{\gamma}{a^2} = \frac{\lambda}{a^{3(1+\alpha)}} \quad , \quad (8)$$

where  $\gamma = \frac{8\pi G}{3}\rho_{0s} - k$  and  $\lambda = \frac{8\pi G}{3}\rho_{0m}$ . The solutions, in terms of  $\gamma$  and  $\lambda$ , expressed in terms of the conformal time  $\eta$ , defined by  $dt = ad\eta$ , are,

- $\alpha = -1$  (vacuum energy)

$$a = -\sqrt{\frac{\gamma}{\lambda}} \frac{1}{\sinh \sqrt{\gamma} \eta} \quad , \quad -\infty < \eta \leq 0 \quad ; \quad (9)$$

- $\alpha = 0$  (incoherent matter):

$$a = \frac{\lambda}{\gamma} \sinh^2 \frac{\sqrt{\gamma}}{2} \eta \quad , \quad 0 \leq \eta < \infty \quad ; \quad (10)$$

- $\alpha = \frac{1}{3}$  (radiation):

$$a = \sqrt{\frac{\lambda}{\gamma}} \sinh \sqrt{\gamma} \eta \quad , \quad 0 \leq \eta < \infty \quad ; \quad (11)$$

- $\alpha = 1$  (stiff matter):

$$a = \left( \frac{\lambda}{\gamma} \right)^{\frac{1}{4}} \sqrt{\sinh(2\sqrt{\gamma}\eta)} \quad , \quad 0 \leq \eta < \infty \quad . \quad (12)$$

In order to perform a perturbative study of these equations, we return back to the original field equations putting  $\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$ , where  $g_{\mu\nu}$  is the background solution and  $h_{\mu\nu}$  is a small perturbation around it. We impose the synchronous coordinate condition  $h_{\mu 0} = 0$ . We retain the tensorial modes only; the spatial behaviour of the perturbed quantities are expressed through normal modes such that  $\nabla^2 Q_{ij} = -n^2 Q_{ij}$ . We obtain the following perturbed equation governing the evolution of gravitational waves:

$$\ddot{h}_{ij} - \frac{\dot{a}}{a} \dot{h}_{ij} + \left[ \frac{n^2}{a^2} - 2 \frac{\ddot{a}}{a} \right] h_{ij} = 0 \quad . \quad (13)$$

The background solutions are of the form  $a = a(r\eta)$ , where  $r$  is a constant. Rewriting this equation in terms of a new conformal time,  $r\eta = \theta$ , and defining  $h_{ij} \propto h(\eta)Q_{ij}$ ,  $Q_{ij}$

being the eigenfunction in the spatial section with constant curvature defined before, the equation (13) becomes,

$$h'' - 2\frac{a'}{a}h' + \left[ \tilde{n}^2 - 2\left(\frac{a''}{a} - \frac{a'^2}{a^2}\right) \right]h = 0 \quad , \quad (14)$$

where  $\tilde{n}^2 = \frac{1}{r^2}(n^2 + 2k)$  and the primes mean now derivatives with respect to  $\theta$ .

After inserting the background solutions, the equation (14) can be in general rewritten in terms of a hypergeometric equation. Its final solution for different phases of the evolution of the Universe are:

- Vacuum energy phase ( $\alpha = -1$ ):

$$h_1 = \sqrt{x^2 - 1} \left[ \frac{1+x}{2} \right]^{-2+\sqrt{1-\tilde{n}^2}} \times \\ {}_2F_1(2 - \sqrt{1 - \tilde{n}^2}, \frac{1}{2} - \sqrt{1 - \tilde{n}^2}, 1 - 2\sqrt{1 - \tilde{n}^2}, \frac{2}{1+x}) \quad , \quad (15)$$

$$h_2 = \sqrt{x^2 - 1} \left[ \frac{1+x}{2} \right]^{-2-\sqrt{1-\tilde{n}^2}} \times \\ {}_2F_1(\frac{1}{2} + \sqrt{1 - \tilde{n}^2}, 2 + \sqrt{1 - \tilde{n}^2}, 1 + 2\sqrt{1 - \tilde{n}^2}, \frac{2}{1+x}) \quad , \quad (16)$$

$$\tilde{n}^2 = \frac{1}{\gamma}(n^2 + 2k) \quad , \quad x = \cosh \sqrt{\gamma} \eta \quad ;$$

- Radiative phase ( $\alpha = \frac{1}{3}$ ):

$$h = \exp(\pm(\sqrt{1 - \tilde{n}^2})\eta) \sinh \eta \quad , \quad \tilde{n}^2 = \frac{1}{\gamma}(n^2 + k) \quad (17)$$

- Material phase ( $\alpha = 0$ ):

$$h_1 = \sqrt{x^2 - 1} \left[ \frac{x+1}{2} \right]^{1+\sqrt{4-\tilde{n}^2}} \times \\ {}_2F_1(-1 - \sqrt{4 - \tilde{n}^2}, \frac{1}{2} - \sqrt{4 - \tilde{n}^2}, 1 - 2\sqrt{4 - \tilde{n}^2}, \frac{2}{1+x}) \quad , \quad (18)$$

$$h_2 = \sqrt{x^2 - 1} \left[ \frac{x+1}{2} \right]^{1-\sqrt{4-\tilde{n}^2}} \times \\ {}_2F_1(\frac{1}{2} + \sqrt{4 - \tilde{n}^2}, -1 + \sqrt{4 - \tilde{n}^2}, 1 + 2\sqrt{4 - \tilde{n}^2}, \frac{2}{1+x}) \quad , \quad (19)$$

$$x = \cosh(\frac{\sqrt{\gamma}}{2}\eta) \quad , \quad \tilde{n}^2 = \frac{4}{\gamma}(n^2 + k) \quad ; \quad (20)$$

- Stiff matter ( $\alpha = 1$ ):

$$h_1 = \sqrt{x^2 - 1} \left[ \frac{x+1}{2} \right]^{\frac{-1+\sqrt{1-4\tilde{n}^2}}{2}} \times \\ {}_2F_1\left(\frac{1-\sqrt{1-4\tilde{n}^2}}{2}, \frac{1-\sqrt{1-4\tilde{n}^2}}{2}, 1-\sqrt{1-4\tilde{n}^2}, \frac{2}{1+x}\right) , \quad (21)$$

$$h_2 = \sqrt{x^2 - 1} \left[ \frac{x+1}{2} \right]^{\frac{-1-\sqrt{1-4\tilde{n}^2}}{2}} \times \\ {}_2F_1\left(\frac{1+\sqrt{1-4\tilde{n}^2}}{2}, \frac{1+\sqrt{1-4\tilde{n}^2}}{2}, 1+\sqrt{1-4\tilde{n}^2}, \frac{2}{1+x}\right) , \quad (22)$$

$$x = \cosh(2\sqrt{\gamma}\eta) , \quad \tilde{n}^2 = \frac{1}{4\gamma} .$$

In these expressions,  ${}_2F_1$  is a hypergeometric function. We may compare the above results with the one fluid case with  $\alpha = -\frac{1}{3}$  and  $k = 0$ . In this case, equation (13) takes the form,

$$\ddot{h} - 2\frac{\dot{h}}{t} + n^2 h = 0 , \quad (23)$$

with the solution

$$h \propto t^{1 \pm \sqrt{1-n^2}} . \quad (24)$$

These solutions exhibit, for small  $n^2$ , growing and decreasing modes. There is a critical value for  $n^2$  over which the perturbations oscillate with increasing amplitude.

### 3 Analysis of the results

The model discussed here represents a closed Universe with the dynamics of an open one. The solutions for gravitational waves are formally the same as those we can find in an expanding Universe with ordinary matter and  $k = -1$ . How we can distinguish, from the analysis of gravitational waves, the open model from the "coasting" one? The main point is the expression for  $\tilde{n}^2$ . For our model,  $\tilde{n}^2 = \frac{1}{r^2}(n^2 + 2)$ , where  $r^2 = \gamma$  (for  $\alpha = -1, \frac{1}{3}$ ),  $\frac{\gamma}{4}$  (for  $\alpha = 0$ ) and  $4\gamma$  (for  $\alpha = 1$ ), while of an open expanding Universe  $\tilde{n} = n^2 - 2$ . Looking at the perturbed solutions found before, we note that for an open Universe there is a critical value for  $n$  above which the perturbations exhibit an oscillatory behaviour, and under which there is growing and decreasing modes. On the other hand, for the

”coasting Universe” this behaviour occurs only for some ranges of value of  $\gamma$  while for values out of this range there is only oscillatory behaviour. For example, in the case  $\alpha = 0$ , there is only oscillatory modes for  $\gamma < 2$ , while for  $\gamma > 2$  we find essentially the same behaviour as in the open Universe. So, a ”coasting” and an open Universe can become hardly indistinguishable for an extremely high density of the stringlike fluid, but they are clearly different if  $\gamma$  is near one.

This different behaviours must lead to a spectrum of gravitational waves perturbations completely different. The complete analysis of this problem is out of the scope of this letter, since it touches many others questions like the initial spectrum of the perturbations and its eventual quantum mechanical origin. But, we can make some qualitative considerations.

The spectrum can be obtained from the distortion on the CMBR, coming from the integrated Sachs-Wolf effect,

$$\frac{\delta T}{T} = \frac{1}{2} \int h'_{ij} e^i e^j d\eta \quad (25)$$

where  $T$  is the temperature in a given direction characterized by the unitary vector  $e^i$ . The main observable quantity is the two point correlation function  $\left\langle \frac{\delta T(e_2^i)}{T} \frac{\delta T(e_1^i)}{T} \right\rangle$  which can be expand into multipolar coefficients through the expression

$$\left\langle \frac{\delta T(e_2^i)}{T} \frac{\delta T(e_1^i)}{T} \right\rangle = \sum_{l=2}^{\infty} c_l P_l(\cos \theta) \quad (26)$$

where  $\theta$  is the angle between two directions of observation  $\cos \theta = e_1^i e_2^i$ . The specific expression for the coefficients  $c_l$  depends if  $k = 1, 0, -1$  [10] and also on the mechanism that generate the perturbations [11].

In any case, however, the expression for the coefficients  $c_l$  depends on the square of the classical solutions for  $h$  found before. Hence, the behaviour of  $h$  is, of course, essential in the determination of the observed quantities. We remark that for small values of the multipole expansion of the two point correlation function, the main contribution comes from the lower order excitations, i.e.,  $n \rightarrow 0$ . We can see that, for  $\gamma = 1$  and  $n^2 = 0$ , an open Universe exhibits a growing mode, while the ”coasting” Universe leads to gravitational perturbations that oscillates with increasing amplitude.

A detailed analysis of the behaviour of the coefficients  $c_l$  may distinguish between the many scenarios describing an expanding Universe. Even if the relative contribution

of density perturbation is more important than gravitational waves, the detection of the polarization of the CMB photons may indicate the specific contribution of the different modes, refining the trial of cosmological scenario. Hence, it is possible that a "coasting" Universe could be distinguished from an expanding open Universe. We hope to present this analysis in the future.

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